Balun Silabs: Analysis and Synthesis

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Contents

1 Introduction

A balun is a circuit that transforms a single-ended signal with respect to ground into a balanced signal via a differential line.

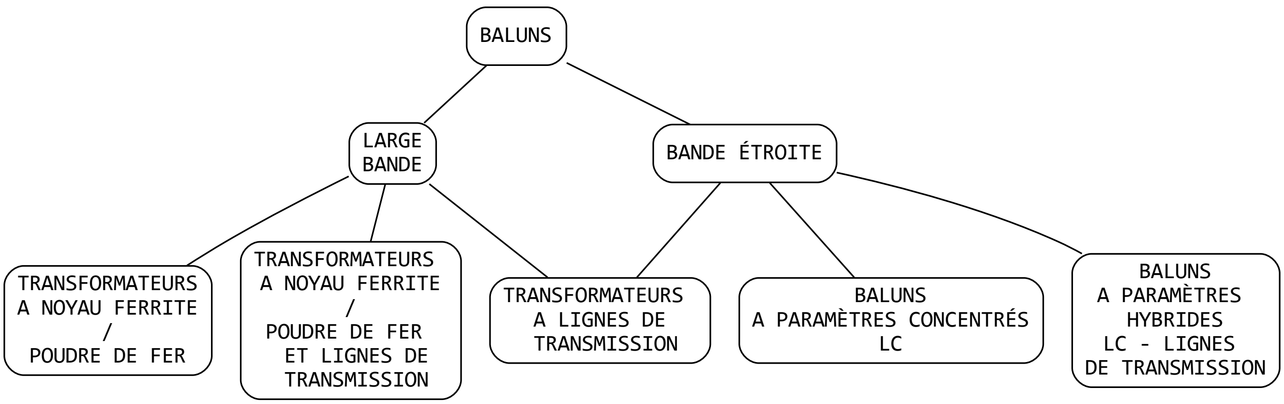


Figure 1: Family of baluns classified by bandwidth.

The purpose of this document is to understand, analyze, and find a method for synthesizing component values to implement Silabs-style "baluns". The development of this study is justified because with a deep understanding of narrow-band balun synthesis, we could subsequently create multi-band evolutions and/or designs with high rejection. The method also allows us to determine the presented impedance as a function of the components used by making a practical measurement or taking into account the common-mode impedance, which is normally a starting parameter: 50Ω.

1.1 Example of a Dual-Band Balun

The dual-band balun was created based on the analysis and deep understanding of the operation of the single-band balun. Below is an example of a balun for 915MHz-1910MHz designed to save switches.

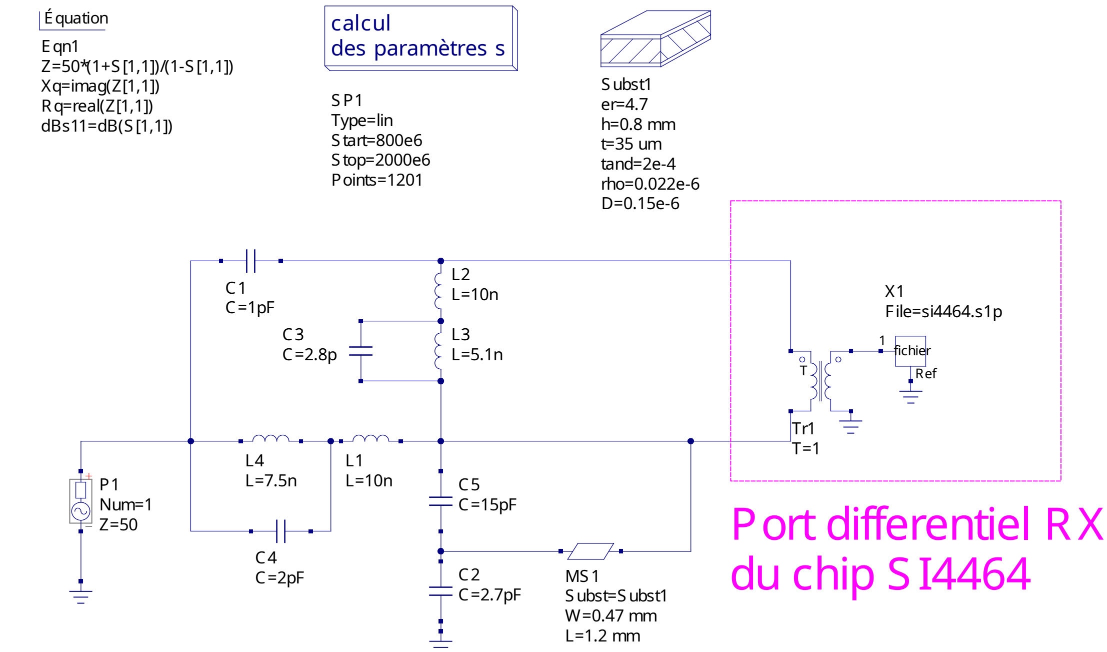


Figure 2: Schematic of the balun for the 915MHz and 1910 MHz bands and the SI4464 chip

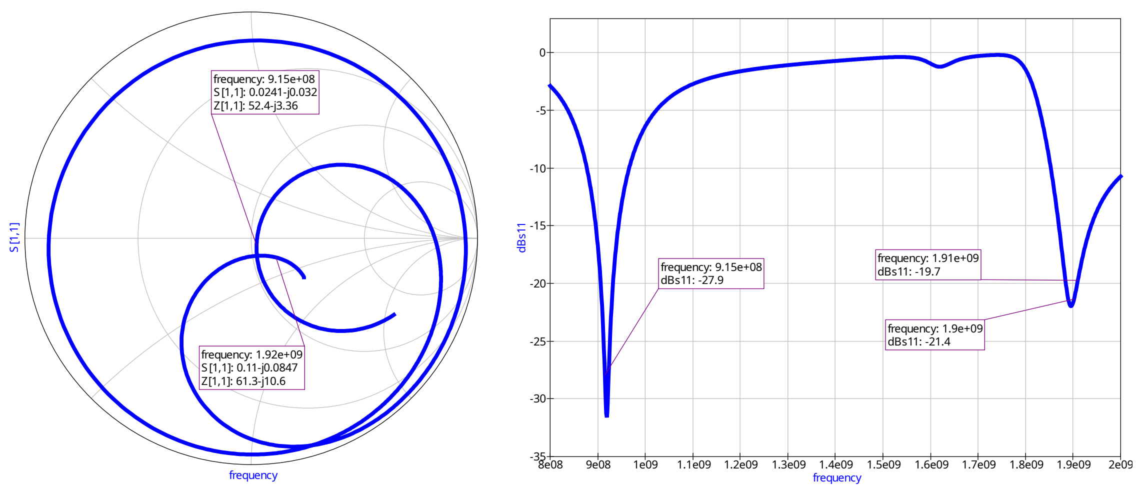


Figure 3: Schematic of the balun for the 915MHz and 1910 MHz bands and the SI4464 chip

2 Classic Schematic of a 4-Element Silabs Balun

In the application note [1], we can find a four-element balun as shown below.

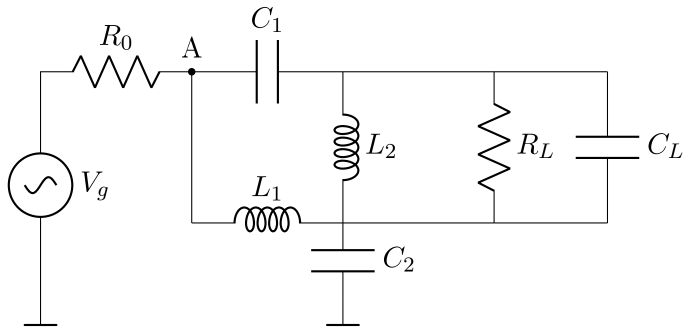
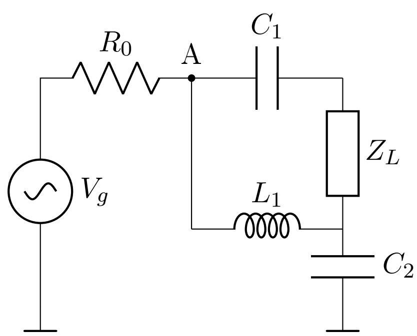


Figure 4: Basic schematic

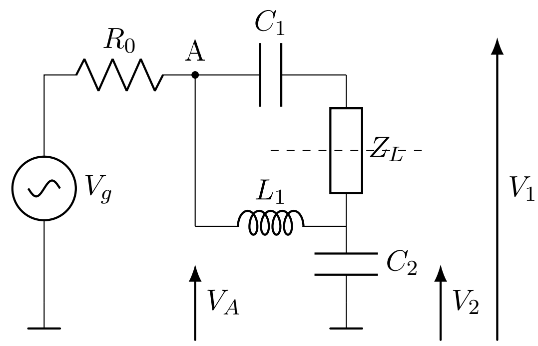
3 Analysis Method

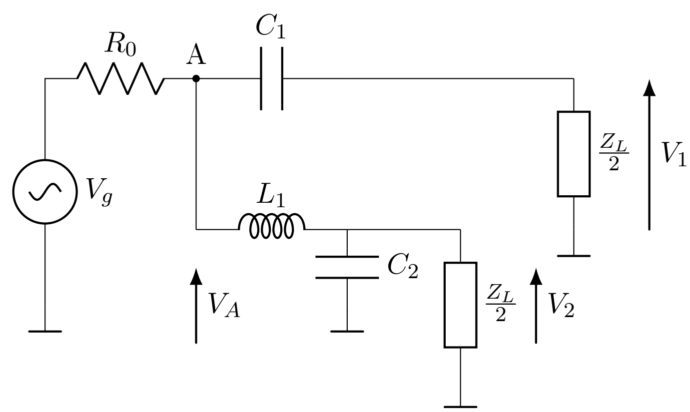
Let R_L and C_L be the classic impedance model in this type of component; to facilitate the analysis, we can combine these two components with L_2 such that:

$$Z_L = jX_{L2}||R_L||jX_{CL} = \frac{jX_T RL}{R_L + jX_T} = \frac{jR_L\left(\omega L_2 - \frac{1}{\omega C_L}\right)}{R_L + j\left(\omega L_2 - \frac{1}{\omega C_L}\right)} = \frac{R_L\left(1-\omega^2 L_2 C_L\right)}{\left(1-\omega^2 L_2 C_L\right) + j\omega R_L C_L} \quad (1)$$



First, by the substitution theorem [3], we can replace the assembly V_g, R_o with a voltage source representing node A. Next, using Bartlett's bisection theorem [2], under differential supply conditions, we can cut the load by symmetry:





Let us impose the conditions of a differential voltage at the two nodes of the load:

$$V_1 = -V_2 \tag{2}$$

The voltages for each of the load halves are:

$$V_1 = V_A \frac{Z_d}{X_{C1} + Z_d} \tag{3}$$

$$V_2 = V_A \frac{Z_d || X_{C_2}}{X_{L2} + X_{C_2} || Z_d} \tag{4}$$

$$V_1 = V_A \frac{Z_d}{\frac{1}{j\omega C_1} + Z_d} = V_A \frac{j\omega C_1 Z_d}{1 + j\omega C_1 Z_d} \tag{5}$$

$$V_2 = V_A \frac{Z_d || \frac{1}{j\omega C_2}}{j\omega L_2 + Z_d || \frac{1}{j\omega C_2}} = V_A \frac{\frac{Z_d}{1+j\omega C_2 Z_d}}{j\omega L_2 + \frac{Z_d}{1+j\omega C_2 Z_d}} = V_A \frac{Z_d}{Z_d(1-\omega^2 L_2 C_2) + j\omega L_2} \tag{6}$$

Following the imposition of opposite voltages:

$$\frac{j\omega C_1 Z_d}{1+j\omega C_1 Z_d} = (-1)\frac{Z_d}{Z_d(1-\omega^2 L_2 C_2)+j\omega L_2} \tag{7}$$

$$j\omega C_1 Z_d \left(Z_d(1-\omega^2 L_2 C_2)+j\omega L_2\right) = (-1)(Z_d + j\omega C_1 Z_d^2) \tag{8}$$
$$j\omega C_1 \left(Z_d(1-\omega^2 L_2 C_2)+j\omega L_2\right) = (-1)(1 + j\omega C_1 Z_d) \tag{9}$$
$$\omega^2 C_1 L_2 + j\omega C_1 Z_d \left(\omega^2 L_2 C_2 - 1\right) - j\omega C_1 Z_d - 1 = 0 \tag{10}$$
$$\left(\omega^2 C_1 L_2 - 1\right) + j\omega C_1 Z_d \left(\omega^2 C_2 L_2 - 2\right) = 0 \tag{11}$$
$$\tag{12}$$

For the expression to be true, both the real and imaginary parts must be set to zero. Hence:

$$\begin{cases} \omega^2 C_1 L_2 - 1 & = 0 \\ \omega^2 C_2 L_2 - 2 & = 0 \end{cases} \tag{13}$$
$$\omega^2 = \frac{1}{L_1 C_1} = \frac{2}{L_1 C_2} \tag{14}$$

With the following result:

$$\boxed{C_2 = 2C_1} \tag{15}$$

3.1 Proof

We will calculate the voltages V_1 and V_2 for each of the halves of the load Z_L respectively, and we will confirm that at the frequency of interest $\omega_o = \frac{1}{\sqrt{L_1 C_1}}$, the voltages V_1 and V_2 will have the same amplitude but will be in phase opposition for any finite impedance/admittance:

$$V_1 = V_A \frac{j\sqrt{\frac{C_1}{L_1}} Z_d}{1 + j\sqrt{\frac{C_1}{L_1}} Z_d} \tag{16}$$

$$V_2 = V_A \frac{Z_d}{Z_d(1 - \frac{L_1 C_2}{L_1 C_1}) + j\frac{L_1}{\sqrt{L_1 C_1}}} = V_A \frac{Z_d}{-Z_d + j\sqrt{\frac{L_1}{C_1}}} = V_A \frac{-j\sqrt{\frac{C_1}{L_1}} Z_d}{1 + j\sqrt{\frac{C_1}{L_1}} Z_d} \tag{17}$$

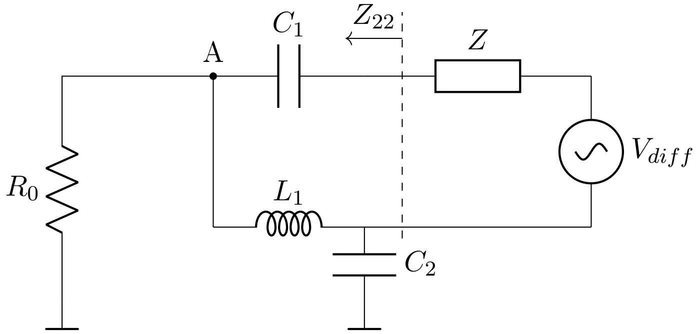
For expression (17), we multiplied the numerator and denominator by $-j\sqrt{\frac{C_1}{L_1}}$. The phase difference of 180° at ω_o is proven for any impedance $Z_d = R_e \pm jX_e$ where $R_e \in [0, +\infty)$, $X_e \in \mathbb{R}$.

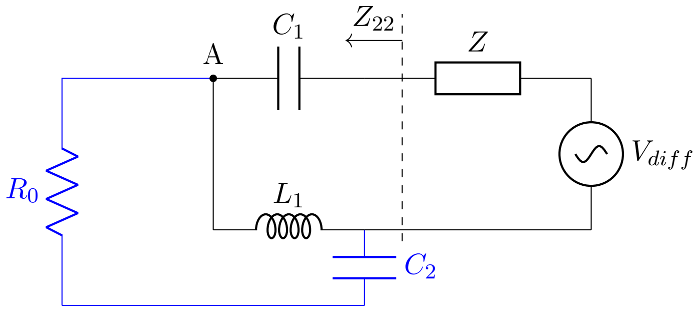
4 Optimal Impedance

We have proven that at the desired frequency, the voltages V_1 and V_2 are phase-shifted by 180°. However, this does not imply that the system is tuned in terms of impedance matching.
To simplify the analysis, we will take advantage of the property of passive networks regarding

symmetrical transfer immittances $m_{12} = m_{21}$. Since port 1 (on the left by convention) is terminated by R_o (the source impedance), which contains a termination with only a real part, we will consider exciting port 2 and determining the impedance $Z_{22}|_{R_o}$ seen from this side, with port 1 loaded by R_o. An optimal impedance matching will be:

$$Z_{opt} = Z_{22}^{*}|_{R_o} \tag{18}$$





$$Z_e(\omega) = X_{C1} + X_{L1}||\left(R_o + X_{C2}\right) \tag{19}$$

$$Z_e(\omega) = \frac{1}{j\omega C_1} + j\omega L_1||\left(\frac{1 + j\omega C_2 R_o}{j\omega C_2}\right) \tag{20}$$

$$Z_e(\omega) = \frac{1}{j\omega C_1} + \frac{j\omega L\left(\frac{1+j\omega C_2 R_o}{j\omega C_2}\right)}{j\omega L_2 + \frac{1+j\omega C_2 R_o}{j\omega C_2}} \tag{21}$$

$$Z_e(\omega) = \frac{1}{j\omega C_1} + \frac{j\omega L_1\left(1 + j\omega C_2 R_o\right)}{\left(1 - \omega^2 L_1 C_2\right) + j\omega C_2 R_o} \tag{22}$$

$$Z_e(\omega) = \frac{\left(1 - 2\omega^2 L_1 C_1\right) - \omega^2 L_1 C_1\left(1 + j\omega 2 R_o C_1 1\right) + j2\omega C_1 R_o}{-\omega^2 C_1^2 2 R_o + j\omega C_1 (1 - \omega^2 2 L_1 C_1)} \tag{23}$$

$$\omega_0 = \frac{1}{\sqrt{C_1 L_1}} \tag{24}$$

$$Z_e(\omega_0) = \frac{(1-2\frac{L_1C_1}{L_1C_1}) - \frac{L_1C_1}{L_1C_1}\left(1 + j2R_o\frac{C_1}{\sqrt{L_1C_1}}\right) + j2Ro\frac{C_1}{\sqrt{L_1C_1}}}{-\frac{C_1^2}{L_1C_1}2R_o + j\frac{C_1}{\sqrt{C_1L1}}(1-2\frac{L_1C_1}{L_1C_1})} \tag{25}$$

$$Z_e(\omega_0) = \frac{-1-1-j2R_o\sqrt{\frac{C_1}{L_1}} + j2R_o\sqrt{\frac{C_1}{L_1}}}{-2R_o\frac{C_1}{L_1} + j\sqrt{\frac{C_1}{L_1}}(-1)} \tag{26}$$

$$Z_e(\omega_0) = \frac{2}{2R_o\frac{C_1}{L_1} + j\sqrt{\frac{C_1}{L_1}}} = \frac{1}{R_o\frac{C_1}{L_1} + \frac{j}{2}\sqrt{\frac{C_1}{L_1}}} \tag{27}$$

$$Z_e(\omega_o) = R_e(\omega_o) + jX(\omega_o) = \frac{R_o\frac{C_1}{L_1} - \frac{j}{2}\sqrt{\frac{C_1}{L_1}}}{R_o^2\left(\frac{C_1}{L_1}\right)^2 + \frac{1}{4}\frac{C_1}{L_1}} \tag{28}$$

$$\begin{cases} R_e(\omega_o) = \frac{R_o\frac{C_1}{L_1}}{R_o^2\left(\frac{C_1}{L_1}\right)^2 + \frac{1}{4}\frac{C_1}{L_1}} = \frac{1}{R_o\left(\frac{C_1}{L_1}\right) + \frac{1}{4R_o}} \\ X_e(\omega_o) = -\frac{\frac{1}{2}\sqrt{\frac{C_1}{L_1}}}{R_o^2\left(\frac{C_1}{L_1}\right)^2 + \frac{1}{4}\frac{C_1}{L_1}} \end{cases} \tag{29}$$

$$R_e = \frac{1}{R_o\left(\frac{C_1}{L_1}\right) + \frac{1}{4R_o}} \rightarrow R_eR_o\frac{C_1}{L_1} + \frac{R_e}{4R_o} = 1 \tag{30}$$

$$R_eR_o\frac{C_1}{L_1} = 1 - \frac{R_e}{4R_o} \tag{31}$$

$$\boxed{\frac{C_1}{L_1} = \frac{4R_o - R_e}{4R_eR_o^2}} \tag{32}$$

Since $\omega_0 = \frac{1}{\sqrt{L_1C_1}}$ we can limit the degrees of freedom :

$$\boxed{L_1 = \frac{2R_o}{\omega_o}\sqrt{\frac{R_e}{4R_o - R_e}}} \tag{33}$$

$$\boxed{C_1 = \frac{1}{2\omega_oR_o}\sqrt{\frac{4R_o}{Re} - 1}} \tag{34}$$

Equations (33) and (34) show that the solution is feasible only if $R_e < 4R_o$. Otherwise, it will be necessary to perform a "pre-matching" to convert R_e to a value lower than $4R_o$. That is, for 50Ω, $R_e < 200\Omega$ (hence the role of L_2, which we will discuss in more detail later).

Now taking the imaginary part from relations (29), we are in a position to express it as a function of the source impedance R_o and the differential real part R_e. We therefore have:

$$Xe = \frac{-\frac{1}{2}\sqrt{\frac{4R_o - R_e}{4R_e R_o^2}}}{R_o^2\left(\frac{4R_o - R_e}{4R_e R_o^2}\right)^2 + \frac{1}{4}\left(\frac{4R_o - R_e}{4R_e R_o^2}\right)} \tag{35}$$

$$\boxed{X_e^* = \frac{R_e^2}{4R_o - R_e}\sqrt{\frac{4R_o}{R_e} - 1} = \frac{R_e}{\sqrt{\frac{4R_o}{R_e} - 1}}} \tag{36}$$

From this, we can see that the optimal impedance should have an inductive component to match the termination of port 1 to R_o. Now, the role of L_2 becomes clearer: on one hand, it serves not only to compensate for the typical capacitive part in the chips, but on the other hand, it also serves to reduce the real part to have $R_e < 4R_o$ if needed, and finally to compensate for X_e. We can therefore distinguish the following cases:

Case	R_L	X_L
1	$R_L \leq 4R_o$	$X_L = 0$
2	$R_L \geq 4R_o$	$X_L = 0$
3	$R_L < 4R_o$	$X_L < X_e \wedge X_L \neq 0$
4	$R_L < 4R_o$	$X_L = X_e \wedge X_L \neq 0$
5	$R_L < 4R_o$	$X_L > X_e \wedge X_L \neq 0$
6	$R_L \geq 4R_o$	$X_L < X_e \wedge X_L \neq 0$
7	$R_L \geq 4R_o$	$X_L = X_e \wedge X_L \neq 0$
8	$R_L \geq 4R_o$	$X_L > X_e \wedge X_L \neq 0$

But these cases can be reduced to:

Case	R_L
1	$R_L \leq 4R_o$
2	$R_L > 4R_o$

Without giving importance to the value of the reactance X_e^*, and then tackling the cancellation of reactances by absorption, like any impedance matching problem.

4.1 Case 1: $R_L < 4R_o$

4.1.1 Sub-case 1.1 $X_L = 0$

This case is the simplest.

1. Calculate L_1 and C_1 using (33) and (34)
2. Calculate X_e^* using (36)
3. Leave L2 unconnected and place an inductor in series with R_L with a value of $L_s = \frac{X_e^*}{\omega_0}$

4.1.2 Sub-case 1.1 $0 < X_L < X_e^*$

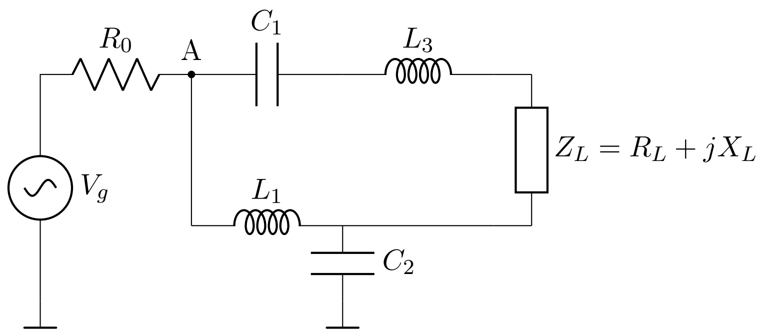
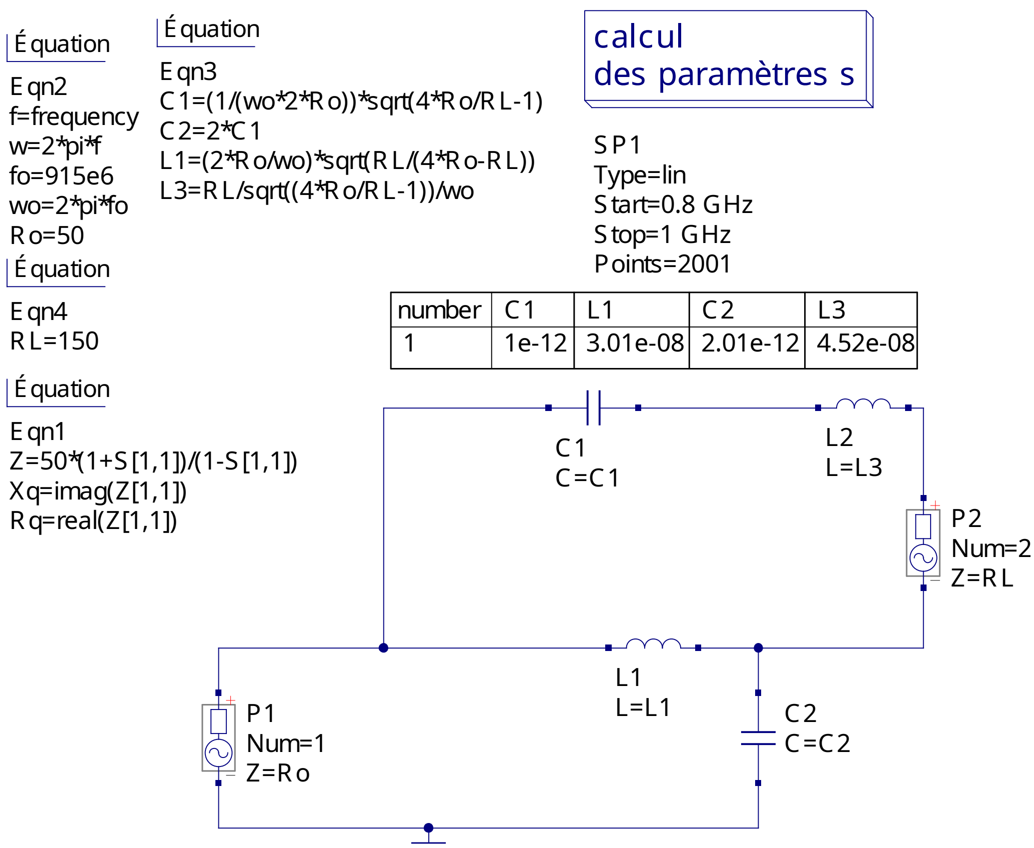


Figure 5: Compensation for lack of reactance by adding an inductor L_3

$$L_3 = \frac{X_e^* - X_L}{\omega_o} \tag{37}$$



number	C1	L1	C2	L3
1	1e-12	3.01e-08	2.01e-12	4.52e-08

Figure 6: Simulation $R_o = 50\Omega$ $R_L = 150\Omega$ $X_L = 0$ $f_o = 915MHz$

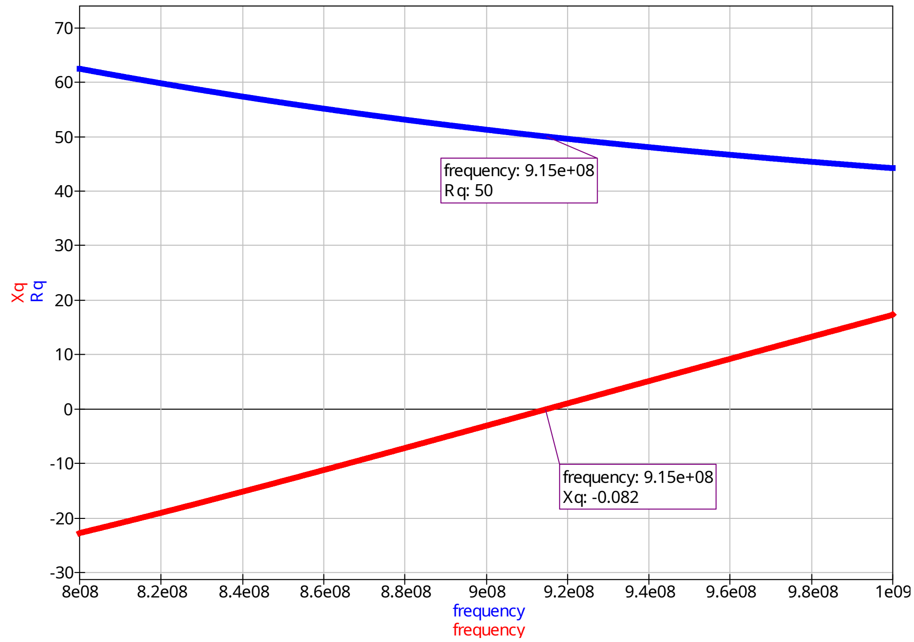


Figure 7: Simulation Results $R_o = 50\Omega$ $R_L = 150\Omega$ $X_L = 0$ $f_o = 915MHz$

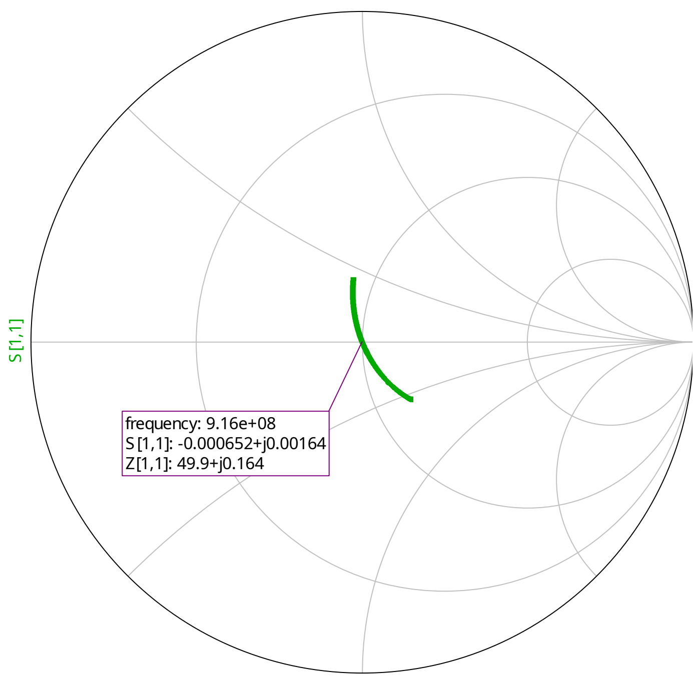


Figure 8: Simulation Results $R_o = 50\Omega$ $R_L = 150\Omega$ $X_L = 0$ $f_o = 915MHz$

Another solution (if physical values allow) is to remove C_1 and decrease the value of L_3:

$$L_3' = \frac{X_e^* - X_L}{\omega_o} - \frac{1}{\omega_o C_1} \tag{38}$$

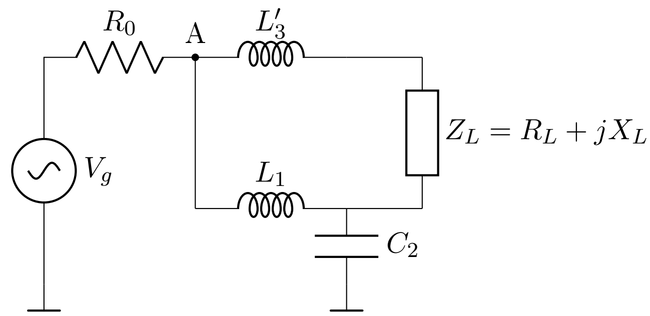


Figure 9: Remove C_1 and perform compensation in L_3

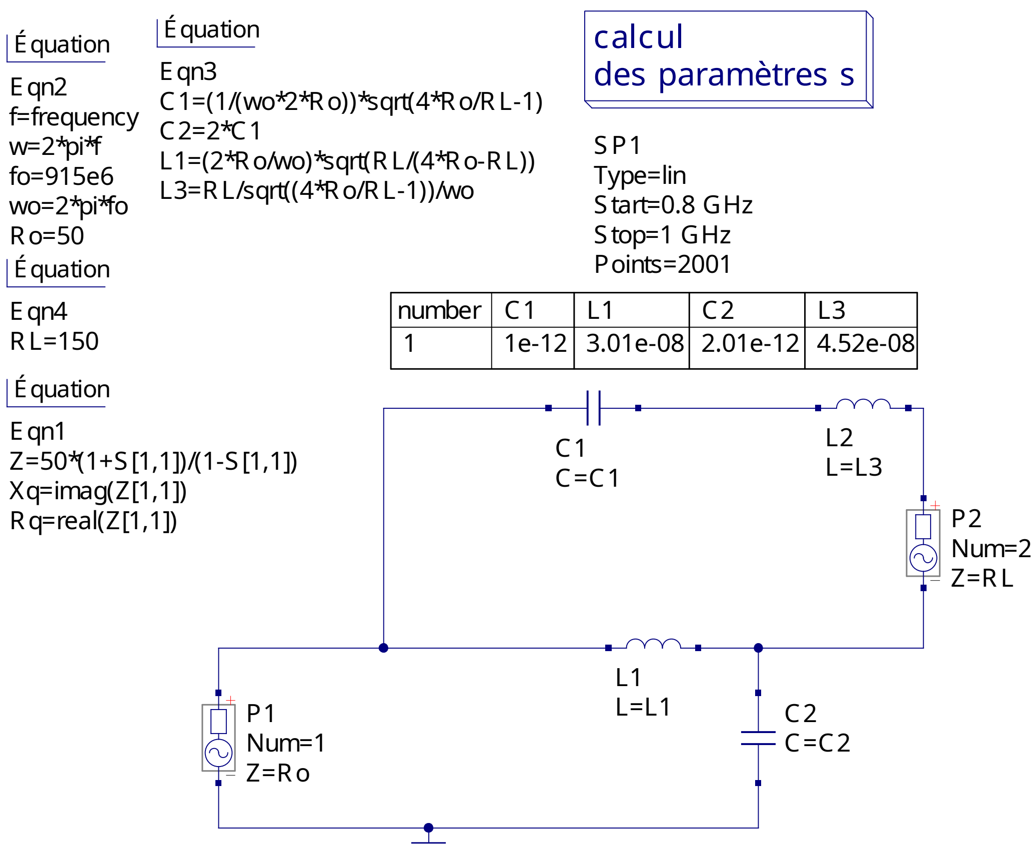


Figure 10: Simulation $R_o = 50\Omega$ $R_L = 150\Omega$ $X_L = 0$ $f_o = 915MHz$

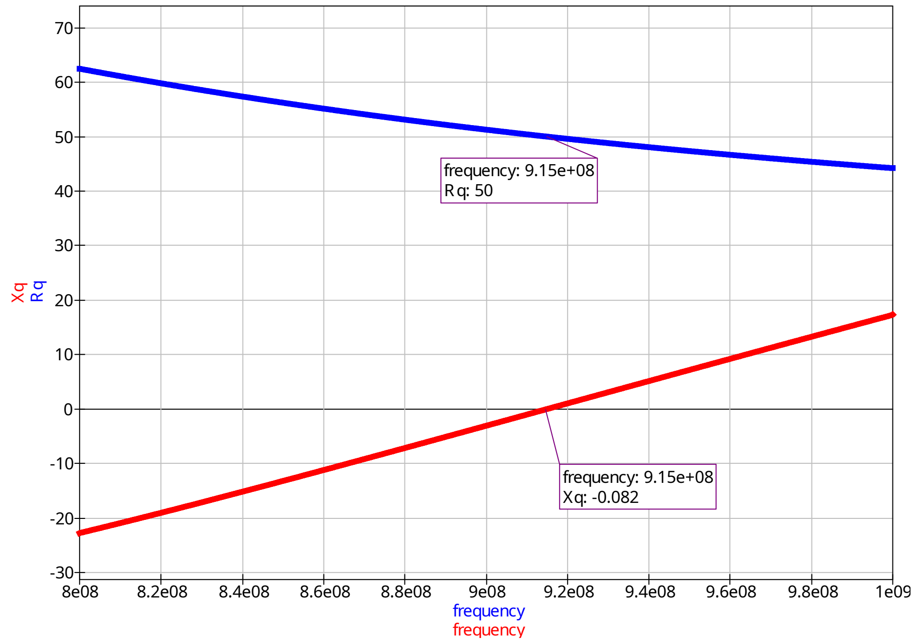


Figure 11: Simulation Results $R_o = 50\Omega$ $R_L = 150\Omega$ $X_L = 0$ $f_o = 915MHz$

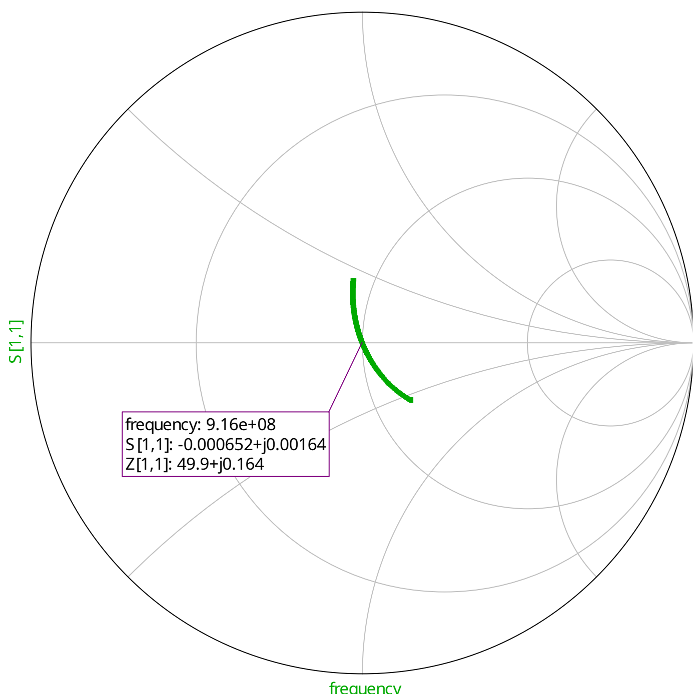


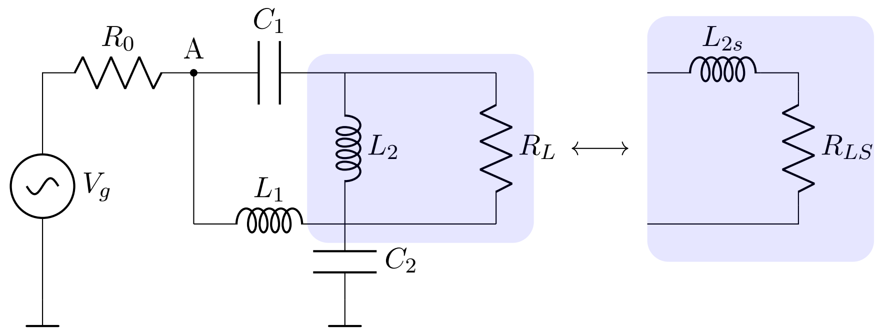
Figure 12: Simulation Results $R_o = 50\Omega$ $R_L = 150\Omega$ $X_L = 0$ $f_o = 915MHz$

4.2 Case 2: $R_L \geq 4R_o$

4.2.1 Sub-case $X_L = 0$

In this case, we are forced to reduce R_L and at the same time add a positive imaginary part. The role of L_2 fits this case well.

1. Match R_L to a value lower than $4R_o$ (we have a certain degree of freedom) with L_2 in parallel.
2. If $X_L > X_e^*$, absorb the excessive reactance by increasing the value of the capacitance C_1.
3. If $X_L < X_e^*$, add a new series inductor or decrease the value of L_2 until $X_L = X_e^*$.



$$R_{LS} = \frac{RL}{1 + \left(\frac{R_L}{\omega_o L_2}\right)^2}$$

$$L_{2S} = \frac{R_L^2}{R_L^2 + (\omega_o L_2)^2} L_2$$

Can we satisfy the condition of matching R_L to R_e with L_2 and, at the same time, matching the imaginary part X_{L2S} to X_e^*? The answer in general is yes.

From the following relations:

$$R_{LS} = \frac{RL}{1 + Q^2} \tag{39}$$

$$R_e = R_{LS} \tag{40}$$

$$Q = \frac{X_e^*}{R_e} = \frac{1}{\sqrt{\frac{4R_o}{Re} - 1}} \tag{41}$$

$$Q = \frac{R_L}{\omega_o L_2} \tag{42}$$

We therefore have:

$$Q^2 = \frac{1}{\frac{4Ro}{R_e} - 1} = \frac{1}{\frac{4Ro}{R_{LS}} - 1} = \frac{1}{\frac{4Ro}{R_{LS}}(Q^2 + 1) - 1} \tag{43}$$

Hence:

$$Q^4 + \left(1 - \frac{R_L}{4R_o}\right) Q^2 - \frac{R_L}{4R_o} = 0 \tag{44}$$

$$Q^2 = \frac{1}{2}\left(\frac{R_L}{4Ro} - 1 \pm \sqrt{\left(1 - \frac{R_L}{4R_o}\right)^2 + \frac{R_L}{R_o}}\right) \tag{45}$$

Since the term under the radical will always be larger than $\frac{R_L}{4R_o} - 1$ and we are restricted to a real and positive result, the $+$ sign must be taken.

$$Q^2 = \frac{1}{2}\left(\frac{R_L}{4Ro} - 1 + \sqrt{1 - \frac{R_L}{2R_o} + \left(\frac{R_L}{4R_o}\right)^2 + \frac{R_L}{R_o}}\right) = \frac{1}{2}\left(\frac{R_L}{4Ro} - 1 + \sqrt{1 + \frac{R_L}{2R_o} + \left(\frac{R_L}{4R_o}\right)^2}\right) \tag{46}$$

$$Q^2 = \frac{1}{2}\left(\frac{R_L}{4Ro} - 1 + \sqrt{\left(1 + \frac{R_L}{4R_o}\right)^2}\right) \tag{47}$$

$$Q^2 = \frac{1}{2}\left(\frac{R_L}{4Ro} - 1 + \left|1 + \frac{R_L}{4R_o}\right|\right) \tag{48}$$

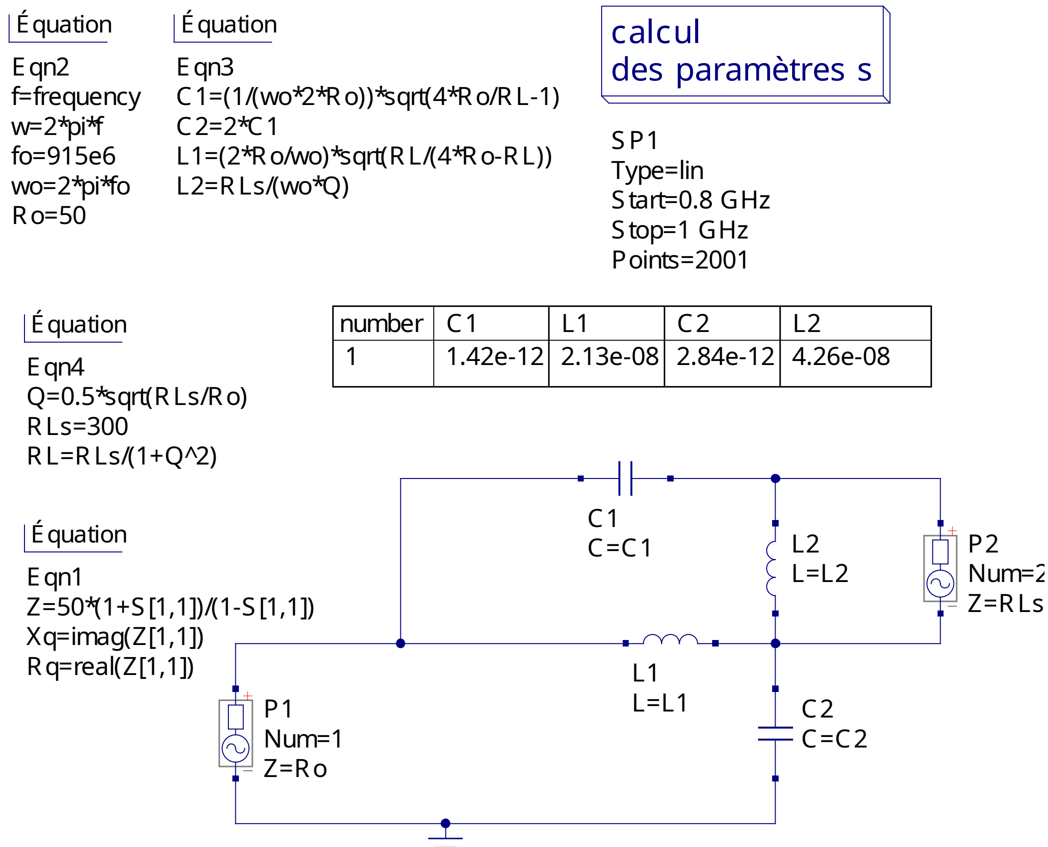
The absolute value can be eliminated because the quantity $1 + \frac{R_L}{4R_o}$ is always positive. We therefore have:

$$Q^2 = \frac{R_L}{4R_o} \tag{49}$$

$$Q = \frac{1}{2}\sqrt{\frac{R_L}{R_o}} \tag{50}$$

Combining relation (50) with (42) gives us the key to obtaining L_2 as a function of $R_o and R_L \geq 4R_o$:

$$\boxed{L_2 = \frac{2}{\omega_o}\sqrt{R_L R_o}} \tag{51}$$



number	C1	L1	C2	L2
1	1.42e-12	2.13e-08	2.84e-12	4.26e-08

Figure 13: Simulation $R_o = 50\Omega$ $R_L = 300\Omega$ $X_L = 0$ $f_o = 915MHz$

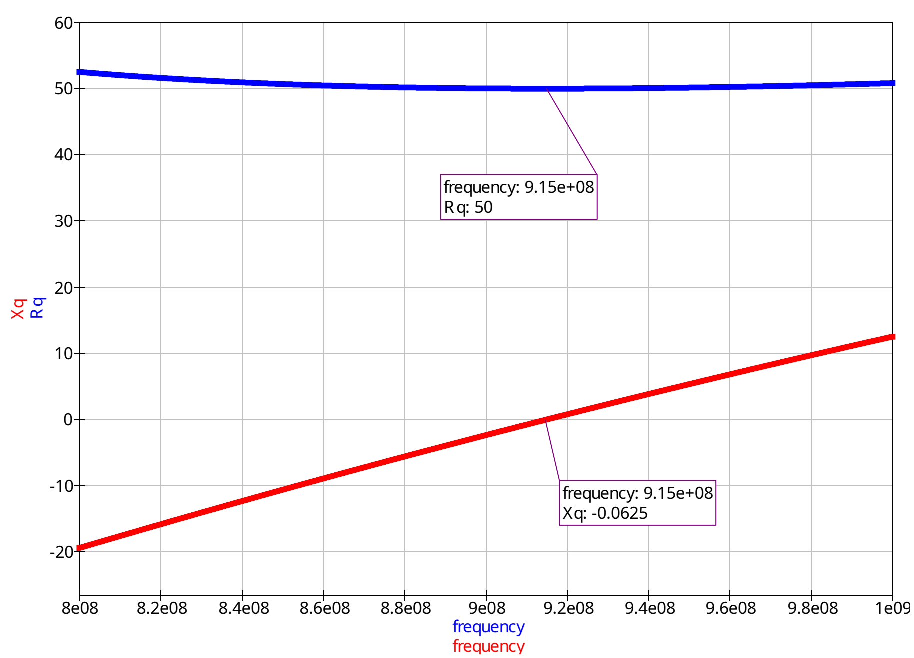


Figure 14: Simulation Results $R_o = 50\Omega$ $R_L = 300\Omega$ $X_L = 0$ $f_o = 915MHz$

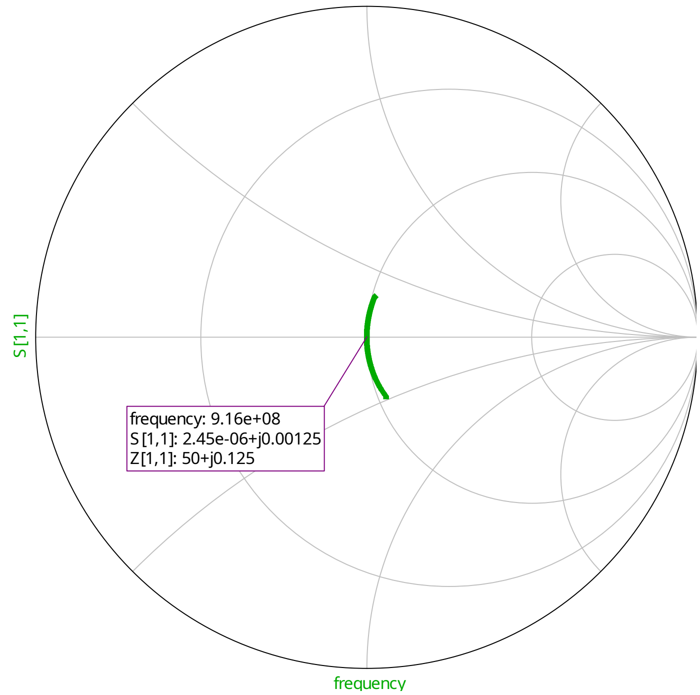


Figure 15: Simulation Results $R_o = 50\Omega$ $R_L = 300\Omega$ $X_L = 0$ $f_o = 915MHz$

4.2.2 Sub-case 2.1 $\omega_o L_{2s} > X_e^*$

To absorb the excessive positive reactance, it must be compensated with a capacitor. We already have C_1, which would need to be modified. The value to compensate for the excessive reactance can be determined as:

$$C_e = \frac{1}{\omega_o \left(\omega_o L_{2S} - X_e^*\right)} \tag{52}$$

Hence:

$$C_1' = \frac{C_1 C_e}{C_1 + C_e} \tag{53}$$

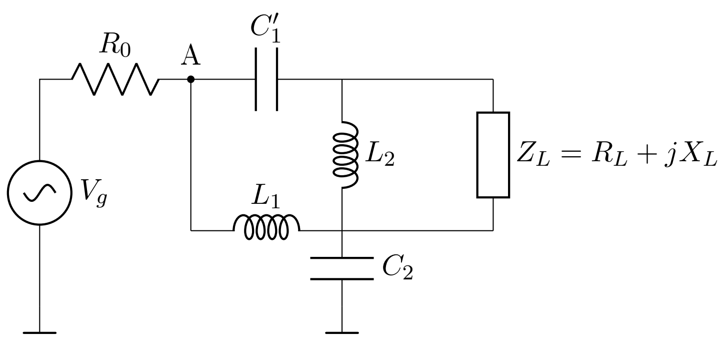


Figure 16: Change the value of C_1 derived from formulas (52) and (53)

4.2.3 Sub-case 2.2 $\omega_o L_{2s} < X_e^*$

For this case, the lack of reactance must be compensated as follows:

$$L_3 = \frac{X_e^*}{\omega_o} - L_{2s} \tag{54}$$

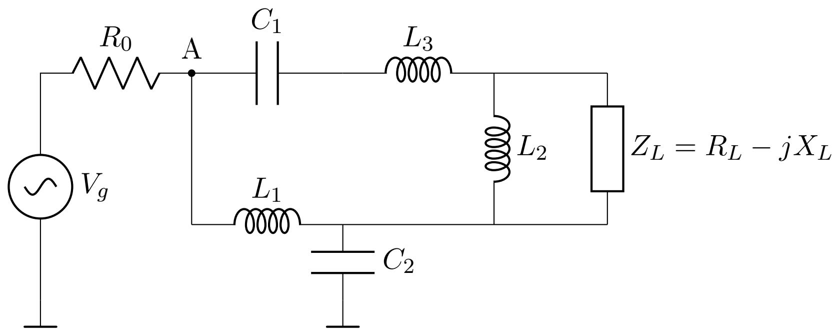


Figure 17: Compensation for lack of reactance by adding an inductor L_3

Alternative solution Instead of adding an inductor L_3, we can also modify the value of C_1 as in Figure 16; the new value is calculated as follows:

$$C_1' = \frac{1}{\omega_o \left(X_e^* - \omega_o L2\right)} + C_1 \tag{55}$$

This latter solution is, in practice, preferable. On one hand, real inductors are generally worse than their capacitive counterparts. On the other hand, we save a component, and furthermore, since the value of C_1 at very high frequencies tends to be quite small, it makes the choice of this value more convenient. Another possibility is shown in Figure 9.

5 A More Abstract Analysis and Dual Version

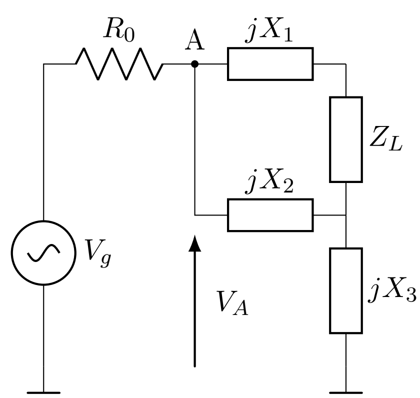


Figure 18: Generic model from which we will derive the relationships between reactances.

$$Z_L = 2Z_d \tag{56}$$
$$V_1 = V_A \frac{Z_d}{Z_d + jX_1} \tag{57}$$
$$V_2 = V_A \frac{Z_d||jX_3}{Z_d||jX_3 + jX_2} = V_A \frac{jZ_dX_3}{jZ_d(X_2 + X_3) - X_2X_3} \tag{58}$$

If we impose:

$$V_1 = -V_2 \tag{59}$$

We have:

$$\frac{1}{Z_d + jX_1} = \frac{-jX_3}{jZ_d(X_2 + X_3) - X_2X_3} \tag{60}$$
$$jZ_d(X_2 + X_3) - X_2X_3 = -jX_3Z_d + X_1X_3 \tag{61}$$
$$jZ_d(X_2 + 2X_3) - X_3(X_1 + X_2) = 0 \tag{62}$$
$$\begin{cases} X_1 + X_2 = 0 \\ X_2 + 2X_3 = 0 \end{cases} \tag{63}$$

Hence:

$$X_2 = -X_1 \tag{64}$$
$$X_3 = \frac{X_1}{2} \tag{65}$$

In conclusion, the reactance X_2 must be of opposite sign to X_1 and X_3; that is, if we use capacitors for X_1 and X_3, X_2 must be an inductor and vice versa. On the other hand, X_3 must be twice X_1. These equations must conform to the desired working frequency/angular frequency. We therefore have for the optimal impedance Z_e^*:

$$Z_e = jX_1 + jX_2||(R_o + JX_3) \tag{66}$$
$$Z_e = jX_1 + \frac{jX_2(R_o + JX_3)}{R_oj(X_2 + X_3)} = \frac{jR_o(X_1 + X_2) - X_1X_2 - X_3(X_1 + X_2)}{R_o + j(X_2 + X_3)} \tag{67}$$
$$Z_e = \frac{jR_o(X_1 - X_1) + X_1^2 - \frac{X_1}{2}(X_1 - X_1)}{R_o + j(\frac{X_1}{2} - X_1)} = \frac{2X_1^2}{2R_o - jX_1} \tag{68}$$
$$Z_e = \frac{4R_o + j2X_1}{\left(\frac{2R_o}{X_1}\right)^2 + 1} \tag{69}$$

After separating the real and imaginary parts:

$$R_e = \frac{4R_o}{\left(\frac{2R_o}{X_1}\right)^2 + 1} \tag{70}$$
$$X_e^* = \frac{-2X_1}{\left(\frac{2R_o}{X_1}\right)^2 + 1} \tag{71}$$

and:

$$\boxed{X_1 = \pm \frac{2R_o}{\sqrt{\frac{4R_o}{R_e} - 1}}} \tag{72}$$

$$\boxed{X_2 = \mp \frac{2R_o}{\sqrt{\frac{4R_o}{R_e} - 1}}} \tag{73}$$

$$\boxed{X_3 = \pm \frac{R_o}{\sqrt{\frac{4R_o}{R_e} - 1}}} \tag{74}$$

$$\tag{75}$$

For $R_e > 4R_o$, X_L is determined as follows. From the relations:

$$Q = \frac{X_1}{2R_o} = \frac{1}{\sqrt{\frac{4R_o}{R_e} - 1}} \tag{76}$$

$$Q = \frac{R_L}{X_L} \tag{77}$$

$$R_e = \frac{R_L}{1 + Q^2} \tag{78}$$

Taking (76) and (78):

$$Q = \frac{1}{\sqrt{\frac{4R_o}{\frac{R_L}{1+Q^2}} - 1}} \rightarrow Q^2 = \frac{R_L}{4R_o(1+Q^2) - R_L} \tag{79}$$

$$4R_oQ^4 + (4R_o - R_L)Q^2 - R_L = 0 \tag{80}$$

$$Q^4 + \left(1 - \frac{R_L}{4R_o}\right)Q^2 - \frac{R_L}{4R_o} = 0 \tag{81}$$

$$Q^2 = \frac{1}{2}\left[\left(\frac{R_L}{4R_o} - 1\right) \pm \sqrt{\left(\frac{R_L}{4R_o} - 1\right)^2 + \frac{R_L}{R_o}}\right] \tag{82}$$

$$Q^2 = \frac{1}{2}\left[\left(\frac{R_L}{4R_o} - 1\right) + \sqrt{\frac{R_L^2}{16R_o^2} - \frac{R_L}{2R_o} + 1 + \frac{R_L}{R_o}}\right] = \frac{1}{2}\left[\left(\frac{R_L}{4R_o} - 1\right) + \sqrt{\left(\frac{R_L}{4R_o} + 1\right)^2}\right] \tag{83}$$

$$Q^2 = \frac{1}{2}\left[\left(\frac{R_L}{4R_o} - 1\right) + \left|\frac{R_L}{4R_o} + 1\right|\right] \tag{84}$$

$$Q = \frac{1}{2}\sqrt{\frac{R_L}{R_o}} \tag{85}$$

Now with (77) and (85):

$$\boxed{X_L = 2\sqrt{R_oR_L}} \tag{86}$$

6 Inverse Characterization

Let us now consider the inverse problem: suppose we can measure the impedance of the single-ended port and want to deduce the impedance of the differential port. If we know the component values, the impedance of the differential port (chip side) can be deduced using "de-embedding" techniques. This can allow us to redesign the system in multiple bands by directly knowing the differential load impedance of the chip without having to modify anything other than the single-ended port.

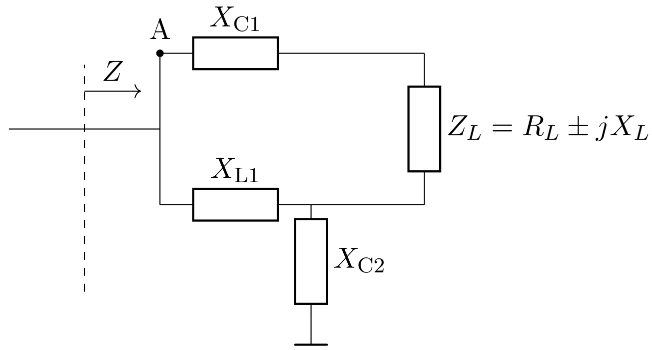


Figure 19: Deduction of Z_L from the opposite port

$$Z = (X_{C1} + Z_L)||X_{L1} + X_{C2} \tag{87}$$

$$(Z - X_{C2}) = \frac{(X_{C1} + Z_L)X_{L1}}{X_{C1} + Z_L + X_{L1}} \tag{88}$$

$$(Z - X_{C2})(X_{C1} + X_{L1}) + Z_L(Z - X_{C2}) = X_{C1}X_{L1} + Z_LX_{L1} \tag{89}$$

$$\boxed{Z = \frac{X_{C1}X_{L1} + (X_{C2} - Z)(X_{C1} + X_{L1})}{Z - (X_{C2} + X_{L1})}} \tag{90}$$

With:

$$X_{C1} = \frac{1}{j\omega_o C_1} \tag{91}$$

$$X_{C2} = \frac{1}{j\omega_o C_2} \tag{92}$$

$$X_{L1} = j\omega_o L_1 \tag{93}$$

$$\tag{94}$$

If we have L_2 in parallel with the chip, we just need to remove it by working with admittances:

$$Y' = \frac{1}{Z} - \frac{1}{X_{L2}} \tag{95}$$

$$Z' = \frac{1}{\frac{1}{Z} - \frac{1}{X_{L2}}} = \frac{ZX_{L2}}{X_{L2} - Z} \tag{96}$$

Example As an example, the simulation below for three frequencies: 868 MHz, 915 MHz, and 2.14 GHz respectively, and a load impedance of 300Ω in differential mode and a balun calculated for the 915 MHz band and $R_o = 50\Omega$. The results show the correct values before and after L2 compensation; before compensation, we have a parallel RL circuit and after compensation, we find a real impedance of 300Ω. The very small residual values in the imaginary part after compensation are solely due to rounding in calculations and limited precision.

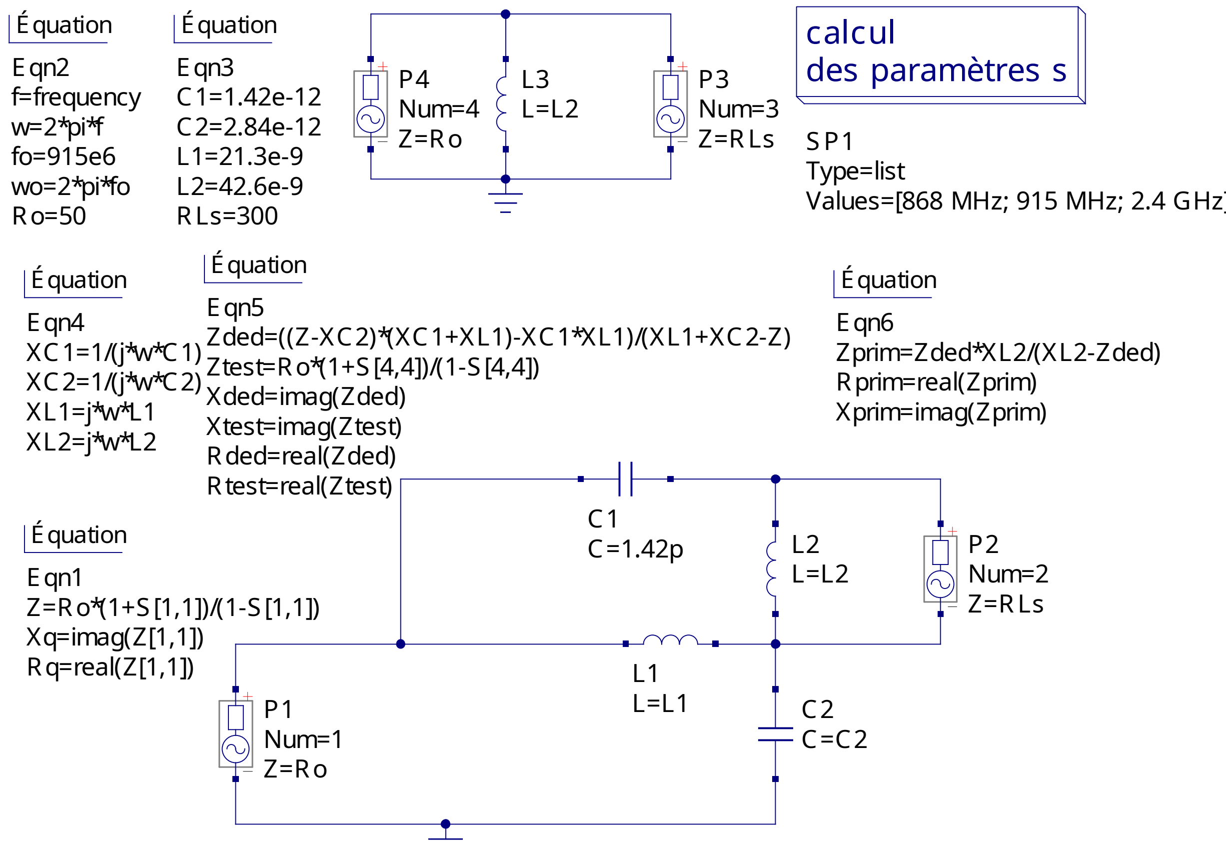


Figure 20: Simulation $R_o = 50\Omega$ $R_L = 300\Omega$ $X_L = 0$ $f_o = 915MHz$

frequency	Rq
8.68e08	50.348
9.15e08	50
2.4e09	119.64

frequency	Rtest	Rded
8.68e08	112.47	112.47
9.15e08	119.98	119.98
2.4e09	246.29	246.29

frequency	Rprim
8.68e08	300
9.15e08	300
2.4e09	300

frequency	Xq
8.68e08	-7.6076
9.15e08	-0.021407
2.4e09	108.63

frequency	Xtest	Xded
8.68e08	145.23	145.23
9.15e08	146.96	146.96
2.4e09	115.02	115.02

frequency	Xprim
8.68e08	-6.07e-14
9.15e08	-1.46e-13
2.4e09	9.06e-14

Figure 21: Simulation Results $R_o = 50\Omega$ $R_L = 300\Omega$ $X_L = 0$ $f_o = 915MHz$

7 Symmetrical Version

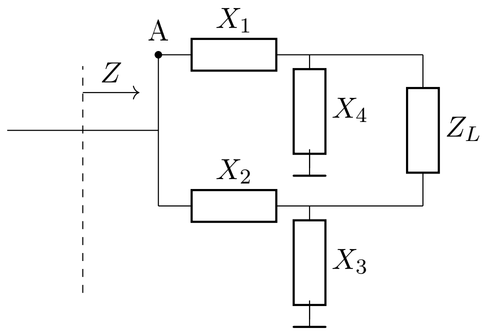


Figure 22: Symmetrical balun

By the same procedure as before:

$$Z_d = \frac{Z_L}{2} \tag{97}$$

$$V_1 = V_A \frac{jX_4||Z_d}{jX_1 + jX_4||Z_d} \tag{98}$$

$$V_2 = V_A \frac{jX_3||Z_d}{jX_2 + jX_3||Z_d} \tag{99}$$

Let us impose the following symmetries:

$$X_1 = X_3 = X_A \tag{100}$$

$$X_2 = X_4 = X_B \tag{101}$$

$$V_1 = -V_2 \tag{102}$$

Hence:

$$V_A \frac{jX_B||Z_d}{jX_A + jX_B||Z_d} = -V_A \frac{jX_A||Z_d}{jX_B + jX_A||Z_d} \tag{103}$$

$$\frac{X_B}{jZ_d(X_A + X_B) - X_A X_B} = \frac{-X_A}{jZ_d(X_A + X_B) - X_A X_B} \tag{104}$$

$$\boxed{X_B = -X_A} \tag{105}$$

The network can be expressed graphically as a lattice network:

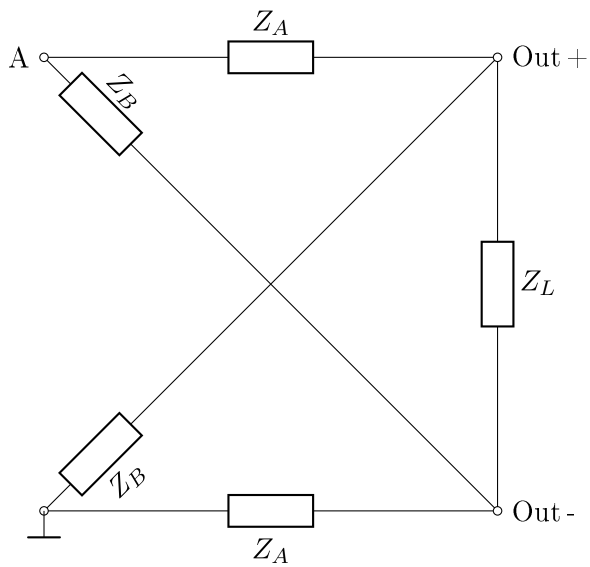


Figure 23: Equivalent Lattice Version

The impedance seen by the input port (on the left) is therefore:

$$Z_o = Z_1||Z_2 \tag{106}$$

$$Z_1 = jX_A + jX_B||Z_d = \frac{jZ_d(X_A + X_B) - X_A X_B}{jX_B + Z_d} = \frac{X_A^2}{Z_d - jX_A} \tag{107}$$

$$Z_2 = jX_B + jX_A||Z_d = \frac{jZ_d(X_A + X_B) - X_A X_B}{jX_A + Z_d} = \frac{X_A^2}{Z_d + jX_A} \tag{108}$$

$$Z_o = \frac{\frac{X_A^2}{Z_d^2 + X_A^2}}{\frac{1}{Z_d + jX_A} + \frac{1}{Z_d - jX_A}} = \frac{X_A^2}{2Z_d} \tag{109}$$

$$\boxed{jX_A = \pm j\sqrt{2Z_d Z_o} = \pm j\sqrt{Z_L Z_o}} \tag{110}$$

$$\boxed{jX_B = \mp j\sqrt{2Z_d Z_o} = \mp j\sqrt{Z_L Z_o}} \tag{111}$$

Let us do an example with $R_o = 50\Omega$, and $Z_L = 200\Omega$ at $f_o = 915\text{MHz}$:

$$X_A \to L_A = \frac{\sqrt{Z_L Z_o}}{\omega_0} \tag{112}$$

$$X_B \to C_B = \frac{1}{\omega_0 \sqrt{Z_L Z_o}} \tag{113}$$

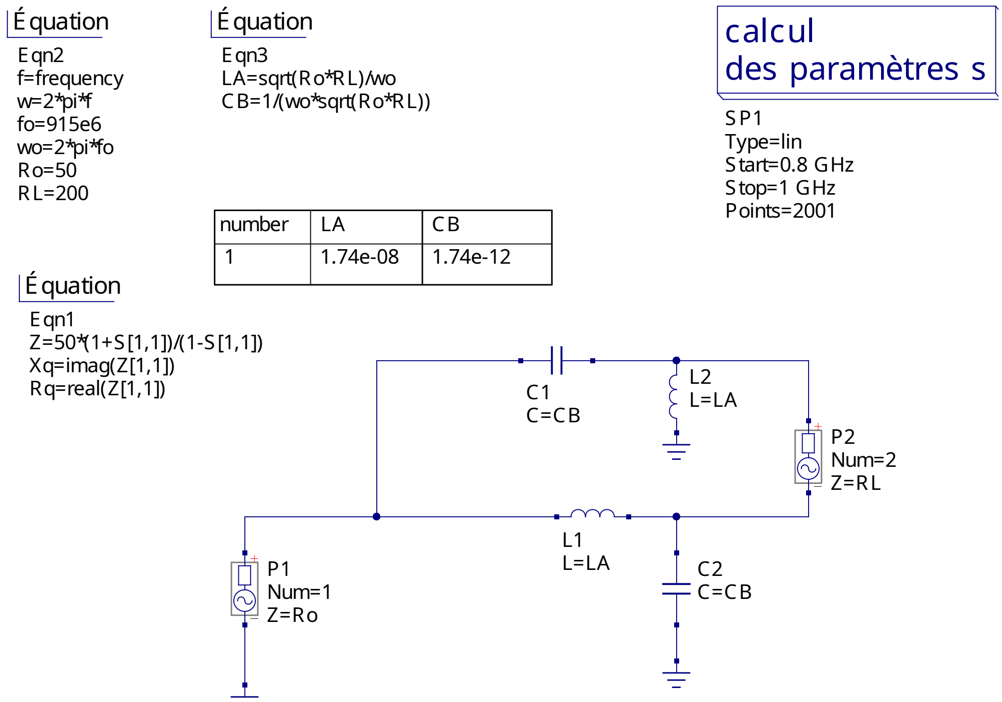


Figure 24: Simulation $R_o = 50\Omega$ $R_L = 200\Omega$ $f_o = 915MHz$, $L_A = 17.4$ nH, $C_B = 1.74$pF

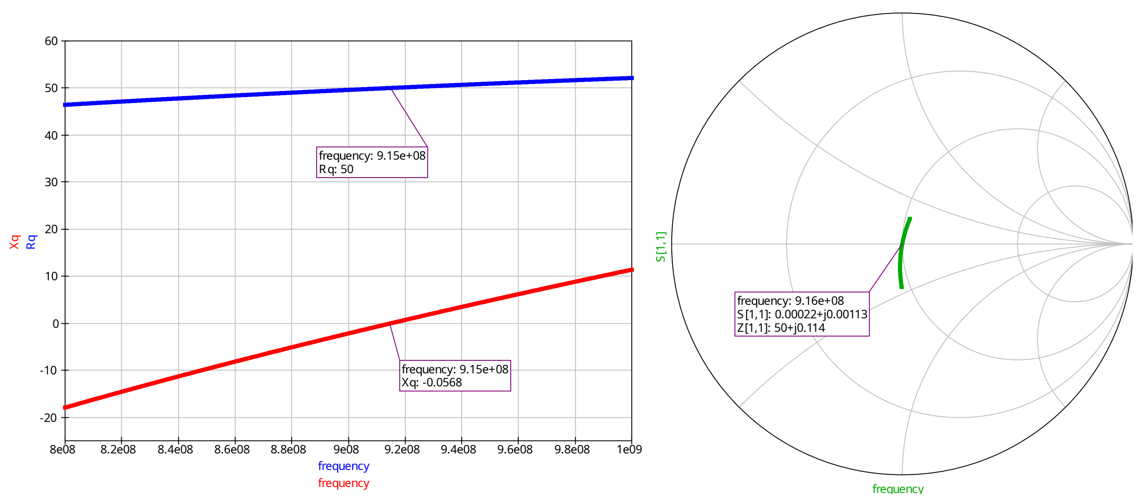


Figure 25: Simulation $R_o = 50\Omega$ $R_L = 200\Omega$ $f_o = 915MHz$, $L_A = 17.4$ nH, $C_B = 1.74$pF

References

[1] Silicon Labs, AN643 *Si446x/Si4362 RX LNA Matching*

[2] Bartlett, AC. *An extension of a property of artificial lines* The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 1927

[3] Millman, Jacob *A Useful Network Theorem*. Proceedings of the IRE. 28 (9): 413–417, 1940